EUSKAL ESTATISTIKA ERAKUNDEA
INSTITUTO VASCO DE ESTADÍSTICA

ECONOMIC ACCOUNTS CHAIN SERIES

1. Introduction

The Quarterly Economic Accounts published for the second quarter of 2005 and the Annual Economic Accounts series incorporate a new means of measurement to substitute the fixed-base rates in volume (constant prices) used up to now.

The methodology applied in this new technique allows us to obtain more precise and much more updated estimations for the components of supply and demand, since the growth of these aggregates in each period will not now be deflated to base year prices, in our case 2000, as has previously been the case. Now, for each period in question, the value of the various aggregates will be expressed in prices from the previous year $(t-1)$. This, in turn, implies that over the length of the series the base year will not be fixed, but that there will be a moving base.

These measures at previous year prices are known as 'links' and the successive production of these links allows us to obtain a time series called a 'Chain Volume Index'.

This series in turn is complemented by another including the monetary value of these indices in euros for a certain year, which is taken as a reference, providing a clearer picture of the measures introduced. This series is known as 'Chain Volume'.

However, this novel form of volume measurement has an important drawback in that there is a loss of additivity between aggregates and their components. In other words, the sum of the supply sectors, or alternatively of the demand aggregates, does not now coincide with the value of GDP. This is a direct consequence of applying the chaining method and should not be interpreted as an error in the data supplied.

The change in methodology carried out on the Quarterly Economic Accounts has its frame of reference in the Commission Decision number $98/715/EC$. The Decision clearly establishes in Part I of its Annex I that the legal act of introducing chain series for the measurement of volume solely concerns the Annual Accounts, which implies that there is no legal obligation to introduce the changes in Quarterly Accounts. However, the Decision also stresses the guiding principle that there should be consistency between the quarterly data and the annual data, so that both Eurostat and other international statistics institutions such as the International Monetary Fund (IMF) or the Organisation for Economic Co-operation and

Development (OECD) urge the various official statistics bodies to also implement chainlinking in their Quarterly Accounts. It is in this context that Eustat has chosen to put its production into practice.

2. Chainlinking annual economic series

The following section gives a simple explanation of the concepts on which the new methodology introduced to produce the Chain Index number series is based, with a parallel numerical example to further aid comprehension. We begin with the case of the series of annual data and go on to extend its application to quarterly data.

Let us take a simple example where we consider an economy which produces solely 2 goods, A and B, for which we have a series with quantities produced and their respective prices.

t	A		B	
0	p_0^A	q_0^A	p_0^B	q_0^B
1	p_1^A	q_1^A	p_1^B	q_1^B
2	p_2^A	q_2^A	p_2^B	q_2^B
$\vdots$	$\vdots$		$\vdots$	
T	p_T^A	q_T^A	p_T^B	q_T^B

Our example starts from the following data for the period $2000 - 2004$:

Table 1

	p_{At}	q_{At}	p_{Bt}	q_{Bt}
2000	$7,0$	$251,0$	$6,0$	$236,0$
2001	$5,5$	$282,0$	$9,0$	$227,0$
2002	$4,0$	$318,0$	$11,5$	$218,0$
2003	$3,0$	$358,0$	$13,5$	$210,0$
2004	$2,6$	$385,0$	$15,4$	$211,0$

To study the evolution of the quantities produced for both goods, a simple series of index numbers can be produced for each article measuring the relation between the level of output in any year t and that of a given year which serves as a reference for the analysis of data and which we will call the 'base year'. The expression for these indices, taking year 0 as the base year would be:

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$$\text{Product A} \quad i^A_{t,0} = \frac{q^A_t}{q^A_0}100$$

$$\text{Product B} \quad i^B_{t,0} = \frac{q^B_t}{q^B_0}100$$

In the numerical example, 2000 is taken as the base year, giving:

Table 2

	A	B
2000	100,0	100,0
2001	112,4	96,2
2002	126,7	92,4
2003	142,6	89,0
2004	153,4	89,4

Given that the series of index numbers are constructed for the base year 2000, the results that they show us should always use this year as a reference. For 2002, therefore, we can see that the quantities produced of product A increased 26,7 % in relation to base year 2000, and those of B decreased 7,6 %. In the same way, in 2004, while product A shows a 53,4 % rise in relation to 2000, B shows a 10,6 % fall.

Similarly, we can see the evolution of the quantities produced jointly of the 2 products, creating a series of compound index numbers, showing the evolution of the aggregate output, which is to say relating the sum of the output of the 2 products for any year t with the sum of the products in the base year 0:

$$Q_{t,0} = \frac{\sum_j q^j_t}{\sum_j q^j_0}100 \qquad \text{donde } j = A, B \text{ products}$$

Table 3

	$\sum q_{jt}$	$Q_{t,2000}$
2000	487	100,0
2001	509	104,5
2002	536	110,1
2003	568	116,6
2004	596	122,4

We can clearly see that, aggregated, given the different results registered in the output of each product, the quantities jointly increased, although only by 22,4 % after 2000.

If what concerns us is studying the evolution in time of the monetary value (or volume) of this production, the variations in the quantities should be combined with the modifications to their respective prices. Multiplying both for each product and period gives us the output value:

t	A	B
0	$p_0^A q_0^A$	$p_0^B q_0^B$
1	$p_1^A q_1^A$	$p_1^B q_1^B$
2	$p_2^A q_2^A$	$p_2^B q_2^B$
$\vdots$	$\vdots$	$\vdots$
T	$p_T^A q_T^A$	$p_T^B q_T^B$

We can see the evolution of the output value with a new series of index numbers that now show how the value varies for year t in relation to the corresponding one for year 0.

$$V_{t,0} = \frac{\sum_j p_t^j q_t^j}{\sum_j p_0^j q_0^j} 100 \qquad j = A, B$$

Table 4

	$p_{At}q_{At}$	$p_{Bt}q_{Bt}$	$\sum p_{jt}q_{jt}$	$V_{t,2000}$
2000	1,757	1,416	3,173	100, 0
2001	1,551	2,043	1,594	113, 3
2002	1,272	2,507	3,779	119, 1
2003	1,074	2,835	3,909	123, 2
2004	991	3,249	4,241	133, 7

If we bear in mind the prices of the transactions made of the quantities produced, we can see that the value of all the aggregate output increased by 33,7 % from 2000.

It is of statistical and economic interest to be able to differentiate the extent to which these value or volume variations are due to the changes in the quantities or, alternatively, to variations in the price structure. To do this, series of index numbers can be defined which isolate the effects of the variations in quantity from the variations in price.

Therefore, the series of Laspeyres compound indices lets us study the evolution of quantities produced, isolating the effect of price changes. For this, Laspeyres estimates the quantities over the whole length of the time series at constant prices from base year 0 and, given this fixed price structure, compares the evolution of the quantities, always in relation to base year 0:

t	A	B
0	$p_0^A q_0^A$	$p_0^B q_0^B$
1	$p_0^A q_1^A$	$p_0^B q_1^B$
2	$p_0^A q_2^A$	$p_0^B q_2^B$
$\vdots$	$\vdots$	$\vdots$
T	$p_0^A q_T^A$	$p_0^B q_T^B$

$$L_{t,0} = \frac{\sum_j p_0^j q_t^j}{\sum_j p_0^j q_0^j} 100 \qquad j = A, B$$

Table 5

	$p_{A,2000}q_{At}$	$p_{B,2000}q_{Bt}$	$\sum p_{j,2000}q_{jt}$	$L_{t,2000}$
2000	1,757	1,416	3,173	100,0
2001	1,974	1,362	3,336	105,1
2002	2,226	1,308	3,534	111,4
2003	2,506	1,260	3,766	118,7
2004	2,695	1,266	1,961	124,8

Thus, we can see that leaving the price structure that existed in 2000 unchanged, the quantities increased 24,8 % up until 2004.

If we compare this result with that obtained in Table 3, which showed the evolution of quantities without taking prices into account, we can see that there are differences compared to the results obtained with Laspeyres, which did take prices into account, although they remained constant. This is due to the fact that the index obtained in Table 3 gives the same weight to all the quantities (all multiplied by 1), while Laspeyres gives a different weight to the quantities of each product, which is given by the price structure existing in 2000.

However, the evolution shown by the Laspeyres is not accurate, nor is it updated. If we observe the initial figures presented in Table 1 it is clear that the price structure for 2000 is completely obsolete by 2004.

The price of product A diminished by $63\,\%$ while that corresponding to product B increased by more than $150\,\%$, so that carrying out a comparison of the evolution of the quantities without taking this fact into account will lead to a bias in the resulting analysis.

The drawback of adopting constant prices for a certain fixed-base year to estimate quantities is that as the existing exchange structures for that year are modified with the passage of time, the analysis that is made loses relevance and accuracy.

Therefore it is more advisable to continuously update the price structure, changing the base year from which the prices used in the evaluation of quantities are taken.

This is the basis of the methodology which is to be introduced with the new series of Chain Volume Index numbers.

Firstly, we need to estimate the quantities corresponding to each period at prices from the previous year:

t	A	B
0	$p_0^A q_0^A$	$p_0^B q_0^B$
1	$p_0^A q_1^A$	$p_0^B q_1^B$
2	$p_1^A q_2^A$	$p_1^B q_2^B$
$\vdots$	$\vdots$	$\vdots$
T	$p_{T-1}^A q_T^A$	$p_{T-1}^B q_T^B$

Logically, in the first year the quantities will be estimated at prices from the same period, since there is no previous year available.

The Laspeyres expression, defined on the idea of a fixed base for prices (constant prices of year 0), is reintroduced and modified so that the base year is moveable, allowing the price structure to be updated continuously. The base year is now always the previous one $t-1$. In this way, we can follow the evolution of quantities in relation to the previous year using the price structure of this previous year as our reference.

$$E_{t,t-1} = \frac{\sum_j p^j_{t-1} q^j_t}{\sum_j p^j_{t-1} q^j_{t-1}} 100 \qquad j = A, B$$

Each one of the index numbers obtained using this Laspeyres index with base $t-1$ is known as a Link ($E_{t,t-1}$).

Table 6

	$p_{A,t-1}q_{At}$	$p_{B,t-1}q_{Bt}$	$\sum p_{j,t-1}q_{jt}$	$E_{t,t-1}$
2000	1,757	1,416	3,173	100,0
2001	1,974	1,362	3,336	105,1
2002	1,749	1,962	3,711	103,3
2003	1,432	2,415	3,847	101,8
2004	1,155	2,849	4,004	102,4

In our example, the calculation of the links is done by estimating the quantities of each year t at prices of the immediately previous year $t-1$, and compared with the quantities of this previous year $t-1$ estimated at their prices in $t-1$, which is to say the base year is now $t-1$. The quotient between every two years gives us each link in the series, which is shown in Table 6.

This moving base year series tells us that from 2000 to 2001 quantities increased 5,1 % using the 2000 price structure, from 2001 to 2002 quantities increased 3,3 % using the 2001 price structure, from 2002 to 2003 they increased 1,8 %, using the 2002 price structure, and so on; however, this series does not allow us to compare non-consecutive years. This is to say that since the estimations have been made by pairs of consecutive years (t in relation to $t-1$), the comparison of non-consecutive years requires the creation of a series that shows a complete homogenous time sequence that is comparable. For this, a year, known as the 'reference year' is defined and assigned the value 100 (as if it were a fixed-base year) and, using this value, the newly-calculated links are 'chainlinked', multiplying each link by the chain accumulated up to the previous year resulting from the same process.

Table 7

t	Link	Chain Index
2000	100,0	100,0
2001	105,1	105,1
2002	103,3	108,6
2003	101,8	110,5
2004	102,4	113,2

By chainlinking the data, the price structure is updated, since each new chain index incorporates the variation undergone by the prices in the last year in question. In this way, we can see that the real variation that the quantities underwent in the period $2000 - 2004$, isolating the effect of prices, was $13{,}2\,\%$, much lower than that calculated by Laspeyres with constant prices from 2000 ($24{,}8\,\%$).

It is important to stress that unlike the Laspeyres Index number series defined for a fixed-base year 0, which also serves as the reference year to study the evolution of the variables, the Chain Index number series does not have a fixed-base year (we have seen that there are as many base years as pairs of consecutive periods being compared), although a reference year is defined to which the value 100 is assigned in order to aid the data analysis.

In addition, in order to simplify the interpretation of data obtained in the form of an index number, a series of monetary terms can be calculated, known as 'Chain Volume', the result of multiplying all the series of indices by the value in current terms observed in the year taken as our reference.

Table 8

	Chain Index	Chain Volume
2000	100,0	3,173
2001	105,1	3,336
2002	108,6	3,445
2003	110,5	3,507
2004	113,2	3,591

As has already been mentioned, the main advantage of this methodology is that it offers much more accurate and updated estimations, since it incorporates a continually updated price structure. On the other hand, this novel means of measurement gives rise to a drawback such as the loss of additivity generated in the monetary series of Chain Volume. This loss of additivity will be reflected, for example, in the sums of supply sectors or, alternatively, in the components of demand, which will now not coincide with the values obtained for GDP (additivity will only be given in the reference year and the one immediately after).

The loss of additivity is intrinsic to the mathematical properties of indices, as we have seen. Therefore, it cannot be said that the quality of the method chosen or the results thus obtained is any other than optimum.

Similarly, if we wish to avoid this loss of additivity in the most recent data, we can always change the reference year by simple rules of three, so that the last data is additive. The change of reference year will alter the values throughout the series of index numbers obtained initially, but the rates of growth remain completely unchanged.

3. Chainlinking quarterly economic series

Adapting the methodology of Chain Indices to quarterly series causes certain special features to be taken into account.

Firstly, we have seen that when calculating the Links, we need to estimate the quantities at prices of the previous period. In the case of quarterly data, this estimation could be made both at previous year prices or prices from a quarter of the previous year (normally the 4th quarter).

During the Eurostat seminar held in October 2002, a consensus was reached whereby the price structure from the previous year would be used to estimate the quantities of each quarter. In fact, Eurostat strongly recommends the use of annual prices from the previous year to estimate quarterly data.

Secondly, and once the estimation is made, comes the phase of chainlinking or overlap and, here too, we have two options open:

- **annual overlap**, in which case the 4 links previously calculated for a particular year t are overlapped or chained, multiplying by the annual Chain Index of the previous year $t-1$. Thus we now obtain 4 quarterly Chain Index numbers, which, on average, coincide with the annual chain (also obtained by multiplying by the previous annual), thereby guaranteeing the consistency between annual and quarterly data recommended by Commission Decision $98/715$. By contrast, the indices calculated using annual overlap might show a considerable leap between the 4th quarter of one year and the 1st quarter of the next, due to the change of year and, with it, the change of the price structure employed.

- **quarterly overlap** (generally the 4th quarter), in which case all the links of year t would be overlapped on the link corresponding to the 4th quarter of $t-1$. In this case, consistency with annual data is not achieved, although it is true that quarterly overlap reduces the leaps in the index series produces between the 4th and 1st quarters.

Both Eurostat and the International Monetary Fund recommend the use of annual overlap for the chainlinking of the series of links. Eustat has chosen the use of annual overlap to guarantee the consistency of quarterly series with those published for annual data.

Following the numerical example presented for the annual data, Table 9 shows the quarterly (and annual) data for the period $2000-2004$:

Table 9

	p_{At}	q_{At}	p_{Bt}	q_{Bt}
2000	7,0	251,0	6,0	236,0
2001T1	6,1	67,4	8,0	57,6
2001T2	5,7	69,4	8,6	57,1
2001T3	5,3	71,5	9,4	56,5
2001T4	5,0	73,7	10,0	55,8
2001	5,5	282,0	9,0	227
2002T1	4,5	76,0	10,7	55,4
2002T2	4,3	78,3	11,5	54,8
2002T3	3,8	80,6	11,7	54,2
2002T4	3,5	83,1	12,1	53,6
2002	4,0	318,0	11,5	218,0
2003T1	3,4	85,5	12,5	53,2
2003T2	3,1	88,2	13,0	52,7
2003T3	2,8	90,8	13,8	52,1
2003T4	2,7	93,5	14,7	52,0
2003	3,0	358,0	13,5	210,0
2004T1	2,6	92,6	14,6	52,6
2004T2	2,6	95,4	15,1	52,6
2004T3	2,4	99,2	15,6	53,4
2004T4	2,7	97,8	16,3	52,4
2004	2,6	385,0	15,4	211,0

Calculating the links requires a prior estimation of the quantities of each quarter at the prices of the previous year and chainlinking with annual overlap entails multiplying the quarterly links to the annual Chain Indices calculated in the previous section.

Table 10

	Valued at $t-1$prices	Links	Chain Indices
2000		100, 0	100, 0
2001T1	817	103, 0	103, 0
2001T2	828	104, 4	104, 4
2001T3	840	105, 8	105, 8
2001T4	851	107, 2	107, 2
2001		105, 1	105, 1
2002T1	917	102, 0	107, 3
2002T2	924	102, 8	108, 1
2002T3	931	103, 6	109, 0
2002T4	939	104, 6	109, 9
2002		103, 3	108, 6
2003T1	954	101, 0	109, 6
2003T2	959	101, 5	110, 2
2003T3	962	101, 9	110, 6
2003T4	972	102, 9	111, 7
2003		101, 8	110, 5
2004T1	975	99, 8	110, 3
2004T2	987	100, 9	111, 6
2004T3	1,011	103, 4	114, 3
2004T4	994	101, 7	112, 4
2004		102, 4	113, 2

The indices thus obtained for the 4 quarters of each year find the average of the annual index obtained previously, which is to say the series are consistent.